Differential Equations

1. What is the order and degree of the following differential equations?

(i) $\quad 5x\left(\dfrac{dy}{dx}\right)^2 - \dfrac{d^2y}{dx^2} - 6y = \log x$ (ii) $\quad x^3\left(\dfrac{d^2y}{dx^2}\right)^2 + \left(\dfrac{dy}{dx}\right)^4 = 0$

2. Determine the order and degree of $t^2\dfrac{d^2s}{dt^2} - st\dfrac{ds}{dt} = s$. Also state if it is linear or non linear.

3. Determine the order and degree of the differential equation: $y = px + \sqrt{a^2p^2 + b^2}, where\ p = \dfrac{dy}{dx}$

4. Find the integrating factor for the following differential equation: $\quad x\log x\dfrac{dy}{dx} + y = 2\log x$

5. Find the differential equation of the family of lines passing through the origin.

6. Find the differential equation of all circles, which pass through the origin and whose centres lie on the Y-axis.

7. Show that the differential equation of which $y = 2(x^2 - 1) + ce^{-x^2}$ is a solution to $\dfrac{dy}{dx} + 2xy = 4x^3$.

8. Form the differential equation of the family of curves represented by the equation:
   $(2x + a)^2 + y^2 = a^2$.

9. Write the degree of the differential equation $\quad x^3\left(\dfrac{d^2y}{dx^2}\right)^2 + \left(\dfrac{dy}{dx}\right)^4 = 0$

10. Solve the following differential equation: $\quad \cos^2 x\dfrac{dy}{dx} + y = \tan x$

11. Solve the following differential equation: $(x^2 - y^2)\,dx + 2xy\,dy = 0$ given that $y = 1$ when $x = 1$

12. Find the particular solution, satisfying the given condition, for the following differential equation:

$$\dfrac{dy}{dx} - \dfrac{y}{x} + \csc\left(\dfrac{y}{x}\right) = 0; y = 0 \text{ when } x = 1$$

13. Find the particular solution of the differential equation satisfying the given conditions:

$x^2dy + (xy + y^2)dx = 0; y = 1 \text{ when } x = 1.$

14. Find the general solution of the differential equation, $\quad x\log x\dfrac{dy}{dx} + y = \dfrac{2}{x}\log x$

15. Solve the following differential equation: $e^x \tan y\,dx + \left(1 - e^x\right)\sec^2 y\,dy = 0$

16. Find the particular solution of the following differential equation:

$(x + 1)\dfrac{dy}{dx} = 2e^{-y} - 1; y = 0 \text{ when } x = 0$

17. Find the particular solution of the differential equation

$\log\left(\dfrac{dy}{dx}\right) = 3x + 4y,$ given that $y = 0$ when $x = 0.$

18. Find the particular solution of the differential equation $x^2dy = (2xy + y^2)\,dx$, given that $y = 1$, when $x = 1$.

19. Find the particular solution of the differential equation, $\left(1 + x^2\right)\dfrac{dy}{dx} = \left(e^{m\tan^{-1}x} - y\right)$ given that $y = 1$ when $x = 0.$

20. Solve the differential equation: $ye^{\frac{x}{y}}dx = \left(xe^{\frac{x}{y}} + y^2\right)dy.\ (y \neq 0)$